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Published in:
Classical and Quantum Gravity

DOI:
10.1088/0264-9381/15/7/002

Class. Quantum Grav. **15** (1998) 1801–1813. Printed in the UK PII: S0264-9381(98)92761-7

D-instantons and asymptotic geometries

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Received 24 March 1998

Abstract. The large-N limit of D-3-branes is expected to correspond to a superconformal field theory living on the boundary of the anti-de Sitter space appearing in the near-horizon geometry. Dualizing the D-3-brane to a D-instanton, we show that this limit is equivalent to a type IIB S-duality. In both cases one effectively reaches the near-horizon geometry. This provides an alternative approach to an earlier derivation of the same result that makes use of the properties of a gravitational wave instead of the D-instanton.

PACS numbers: 1125, 0450, 0465

1. Introduction

Ten years ago a relation was suggested between branes, singletons and anti-de Sitter (adS) spacetime in the context of the 'brane living at the end of the universe' programme [1–4]. In this approach a brane is considered in an adS background§ with the worldvolume of the brane positioned at the boundary of the adS spacetime. The suggestion was made that the degrees of freedom of this brane are the singleton representations of the adS group and products thereof. These so-called singletons do not correspond to local degrees of freedom in the bulk, but instead describe boundary degrees of freedom [5]. At the same time the anti-de Sitter isometry group of the bulk manifests itself as a conformal group on the boundary of the adS spacetime. It therefore seems that the physics of the brane is determined by a conformal field theory defined on the adS boundary.

Recently, there has been renewed interest in these interconnections from different points of view [6–14]. One of the observations is that the anti-de Sitter spacetime also occurs in the (non-singular) near-horizon geometry of the 10-dimensional D-3-brane and the 11-dimensional M-2-brane and M-5-brane [15]. Recently, it has been suggested that the large-N limit of D-3-branes corresponds to a superconformal field theory living on the boundary of this anti-de Sitter space [7]. Moreover, it was observed [9, 10] that via a series of duality transformations the M-2-, D-3- and M-5-branes (with flat asymptotic geometry) can be locally‖ transformed into a non-flat geometry of the type $AdS_4 \times S_7$, $AdS_5 \times S_5$ and $AdS_7 \times S_4$, respectively. These geometries are exactly the (non-singular) near-horizon geometries of the original brane. Notice that the duality transformations have changed the asymptotic geometry. These results are another hint that the physics of these branes are

§ Such backgrounds are natural to consider since they occur in the spontaneous compactification of supergravity theories [1, 2].
‖ The global validity of these duality transformations should be taken with caution [10], see also section 6.

described by supersingleton field theories [10]. It is the purpose of this paper to give an alternative derivation of the duality symmetries relating branes to adS spaces.

The basic idea of [9, 10] is to start from a brane solution (with p spacelike isometries) described by a harmonic function on the transverse space:

$$H = h + \frac{Q}{r^{7-p}}, \qquad r^2 = x_{p+1}^2 + \cdots + x_9^2. \tag{1}$$

Here h is an integration constant and Q represents the charge of the brane. In order to have asymptotic flat geometries we will restrict ourselves to p-branes with $p < 7$. One first relates the brane solution, via U-duality, to a gravitational wave solution

$$\mathrm{d}s_{10}^2 = \mathrm{d}u\,\mathrm{d}v + H\,\mathrm{d}u^2 + \mathrm{d}x_i^2, \qquad i = 2, \ldots, 9, \tag{2}$$

where (u, v) are lightlike coordinates parametrizing a two-dimensional subspace, with signature $(1, 1)$, of the 10-dimensional spacetime and H is a harmonic function of the eight-dimensional transverse space. One next makes a change of coordinates that amounts to an $SL(2, \mathbb{R})$ rotation in the (u, v) space given by

$$\begin{pmatrix} v \\ u \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -h \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v \\ u \end{pmatrix}. \tag{3}$$

After this coordinate transformation one ends up with the same gravitational wave solution, but with the constant h in the harmonic function H set equal to zero. Finally, one dualizes the wave back to the brane solution one started from. The net effect of this web of dualities is that one obtains the same brane solution, but with the constant in the harmonic set equal to zero. This new solution describes exactly the same geometry that one obtains upon approaching the horizon of the original brane solution at $r = 0$ since in that limit one can effectively ignore the constant part in the harmonic function.

Another way of shifting away the constant part of the harmonic function has been discussed in [16]. The basic idea here is to relate the brane to a Kaluza–Klein (KK) monopole instead of a gravitational wave. One next considers the four-dimensional Taub-NUT space of the KK monopole. A TST duality transformation† removes the constant part in the harmonic and one ends up with an Eguchi–Hanson instanton. Dualizing back to the original brane leads to the same result as above.

In this paper we want to consider another intermediate solution which has the advantage that it has a manifest $SL(2, \mathbb{R})$ duality symmetry and that the process of shifting away the constant part of the harmonic functions has a simple interpretation as performing a special $SL(2, \mathbb{R})$ transformation. Since type IIB superstring theory has a manifest $SL(2, \mathbb{R})$ symmetry, it is natural to consider a IIB brane. In fact, the most natural one to consider is the one with the highest-dimensional transverse space which is the D-instanton [17]. We therefore propose to use the D-instanton as an intermediate solution.

As explained in [18] the D-instanton can be understood as a compactified 12-dimensional wave. The metric of such a gravitational wave is given by‡

$$\mathrm{d}s_{12}^2 = \mathrm{d}u\,\mathrm{d}v + H\,\mathrm{d}u^2 + \mathrm{d}s_E^2, \tag{4}$$

where (u, v) are lightlike coordinates parametrizing a two-dimensional torus with $(1, 1)$ signature and fixed volume. The function H is a harmonic function of the Euclidean 10-dimensional space with metric $\mathrm{d}s_E^2$. As shown in [18] reducing the wave (4) over (u, v) yields the D-instanton solution. From this 12-dimensional point of view the special $SL(2, \mathbb{R})$ transformation that transforms away the constant part of the harmonic function

† This duality transformation has the same effect as an Ehlers transformation [16].

‡ The metric is in an (12-dimensional) Einstein frame. Note that there is no dilaton in 12 dimensions.

describing the D-instanton corresponds to an $SL(2, \mathbb{R})$ rotation in the (u, v) space as given by (3). The difference with the approach of [9, 10] is that in that case the $SL(2, \mathbb{R})$ rotation is performed on a two-dimensional subspace of the 10-dimensional spacetime. Here, however, we rotate the two additional dimensions arising in a 12-dimensional interpretation of type IIB superstring theory [19, 20]. This rotation can be interpreted as a special $SL(2, \mathbb{R})$ duality transformation of the type IIB superstring theory.

In the next section we first review the D-instanton solution. In section 3 we describe the web of dualities that makes use of the D-instanton. The supersymmetry of the different configurations, before and after duality, is considered in section 4, while the extension to intersecting configurations is discussed in section 5. A further discussion and interpretation of our results can be found in the conclusions.

2. The D-instanton

In this section we review the properties of the D-instanton solution [17] of IIB supergravity [21]. Since all gauge fields vanish for this solution we only consider the Ramond/Ramond (RR) pseudo-scalar ℓ, the dilaton ϕ and the metric $g_{\mu\nu}$. Introducing the complex scalar S via

$$S = \ell + \mathrm{i}\mathrm{e}^{-\phi}, \tag{5}$$

the Minkowskian IIB action in the Einstein frame is given by

$$\begin{aligned} S &= \int \mathrm{d}^{10}x\,\sqrt{|g|}\left[R + \frac{1}{2}\frac{\partial S\partial\bar{S}}{(\mathrm{Im}\,S)^2}\right] + S_{\partial M} \\ &= \int \mathrm{d}^{10}x\,\sqrt{|g|}\big[R + \tfrac{1}{2}\big(\mathrm{e}^{2\phi}(\partial\ell)^2 + (\partial\phi)^2\big)\big] + S_{\partial M}, \end{aligned} \tag{6}$$

where $S_{\partial M}$ is a boundary contribution that will be discussed in the conclusions. After performing a Wick rotation to Euclidean space the action reads

$$\begin{aligned} S_E &= \int \mathrm{d}^{10}x\,\sqrt{|g|}\left[-R + \frac{\partial S_+\partial S_-}{\frac{1}{2}(S_+ - S_-)^2}\right] + S_{\partial M} \\ &= \int \mathrm{d}^{10}x\sqrt{|g|}\big[-R + \tfrac{1}{2}\big(\mathrm{e}^{2\phi}(\partial\ell)^2 - (\partial\phi)^2\big)\big] + S_{\partial M}, \end{aligned} \tag{7}$$

where the two real scalars $S_\pm$ are defined as

$$S_\pm = \ell \pm \mathrm{e}^{-\phi}. \tag{8}$$

Note, that the kinetic term of ℓ has changed its sign, due to the fact that ℓ is a pseudo-scalar, which under a Wick rotation† transforms as $\ell \to \mathrm{i}\ell$.

The equations of motion corresponding to the Euclidean action (7) are given by

$$\begin{aligned} R_{\mu\nu} &= \mathrm{e}^{2\phi}\partial_\mu\ell\partial_\nu\ell - \partial_\mu\phi\partial_\nu\phi, \\ 0 &= \partial_\mu\big(\sqrt{|g|}g^{\mu\nu}\mathrm{e}^{2\phi}\partial_\nu\ell\big), \\ 0 &= \mathrm{e}^{2\phi}(\partial\ell)^2 + \frac{1}{\sqrt{|g|}}\partial_\mu\big(\sqrt{|g|}g^{\mu\nu}\partial_\nu\phi\big). \end{aligned} \tag{9}$$

† Both the Wick rotation and the parity transformation can be viewed as special cases of a continuous phase transformation, as has been discussed in [22]. From this point of view, the Wick rotation can be seen as the square root of a parity transformation and therefore ℓ transforms as $\ell \to \mathrm{i}\ell$.

We now search for solutions to these equations of motion. Following [17] we assume that the Einstein metric describing the 10-dimensional space transverse to the D-instanton is flat, i.e.

$$\mathrm{d}s^2 = \mathrm{d}r^2 + r^2\,\mathrm{d}\Omega_9, \tag{10}$$

where we have used spherical coordinates†. Under this assumption the general solution to the equations of motion (9) is given by

$$\pm\ell + \alpha = \mathrm{e}^{-\phi} = \frac{1}{H}, \tag{11}$$

where α is constant and H is a general harmonic function over the 10-dimensional flat Euclidean space, i.e. $\partial^2 H = 0$. For the spherical symmetric case this harmonic function is given by

$$H = h + \frac{Q}{r^8}, \tag{12}$$

where h is an integration constant and Q is the Noether charge defined by [17]

$$Q = \pm\frac{1}{8\Omega_9}\int_{\partial M} \mathrm{e}^{2\phi}\,\partial\ell, \tag{13}$$

where $\Omega_9 = 2\pi^{5/2}/24$ is the volume of the 9-sphere. Therefore, in this case the D-instanton solution is parametrized by the three constants α, h and Q.

In the string frame the D-instanton solution (11) reads

$$\begin{aligned} &\pm\ell + \alpha = \mathrm{e}^{-\phi} = H^{-1}, \\ &\mathrm{d}s^2 = \sqrt{H}\left[\mathrm{d}r^2 + r^2\,\mathrm{d}\Omega_9\right] = \left(hr^4 + \frac{Q}{r^4}\right)^{1/2}\left[\left(\frac{\mathrm{d}r}{r}\right)^2 + \mathrm{d}\Omega_9\right]. \end{aligned} \tag{14}$$

Note that the solution is symmetric under the interchange

$$hr^4 \leftrightarrow \frac{Q}{r^4}. \tag{15}$$

It corresponds to a wormhole connecting two asymptotic flat regions (see figure 1). The minimal diameter d_{min} of the wormhole throat equals

$$d_{min}^8 = 32^2 hQ \tag{16}$$

and it is positioned at a value $r = r_{min}$ given by

$$r_{min}^8 = \frac{Q}{h}. \tag{17}$$

Under the 'mirror' symmetry (15) the asymptotically flat regions at $r = 0$ and $r = \infty$ are mapped onto each other, while at the same time Q and h get interchanged. Notice however that, although the metric is symmetric under this 'mirror' symmetry, the dilaton is not. In the $r = \infty$ vacuum the dilaton is finite, whereas it diverges for $r \to 0$. The asymptotic geometry at $r = 0$ is given by a flat spacetime with metric (in string frame)

$$\mathrm{d}s^2 = \sqrt{\frac{Q}{r^4}}\left[\left(\frac{\mathrm{d}r}{r}\right)^2 + \mathrm{d}\Omega_9\right] = \mathrm{d}\rho^2 + \rho^2\,\mathrm{d}\Omega_9, \tag{18}$$

where $\rho = Q^{1/4}/r$.

† The results of [13, 23] suggest that there exists a more general class of D-instanton solutions which can be obtained by replacing the metric $\mathrm{d}\Omega_9$ of the 9-sphere in (10) by the metric $\mathrm{d}s^2_{\mathrm{comp}}$ describing the geometry of any Einstein space that arises in the compactification of (Euclidean) IIB supergravity from 10 to 1 dimensions. The one-dimensional space has an $\mathbb{R}^+$ topology and is parametrized by the radial coordinate r. We will not consider this possibility further in this work and restrict ourselves to the standard D-instanton with flat transverse space.

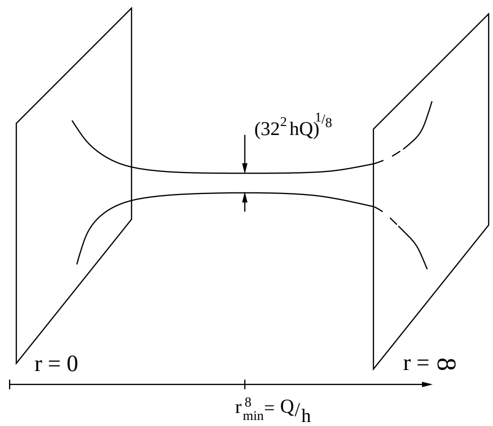

Figure 1. The D-instanton geometry. The D-instanton solution comprises two asymptotic flat regions, one at $r=\infty$ and one at $r=0$ which are connected by a throat with minimal diameter $d_{min}=\left(32^2 hQ\right)^{1/8}$ at position $r_{min}=(Q/h)^{1/8}$.

The equations of motion (9) are invariant under the $SL(2,\mathbb{R})$ transformations

$$S_\pm \to \frac{aS_\pm + b}{cS_\pm + d}, \qquad ad-bc=1, \tag{19}$$

with the two generators

$$\Omega_1\colon \quad S_\pm \to S_\pm + 1, \qquad \Omega_2\colon \quad S_\pm \to -\frac{1}{S_\pm}. \tag{20}$$

Using these transformations we can change the parameters α and h arbitrarily, characterizing the D-instanton solution but *not* Q. In particular, we can transform the constant part h of the harmonic function (12) to zero. Taking the positive sign in (11) this is achieved by the special $SL(2,\mathbb{R})$ transformation†

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \Omega_1^{-\alpha}\Omega_2^{-1}(\Omega_1)^{h/2}\Omega_2\Omega_1^{\alpha} = \begin{pmatrix} 1+\frac{1}{2}h\alpha & \frac{1}{2}\alpha^2 h \\ -\frac{1}{2}h & 1-\frac{1}{2}\alpha h \end{pmatrix}, \tag{21}$$

yielding the solution

$$\ell + \alpha = \mathrm{e}^{-\phi} = \frac{r^8}{Q}. \tag{22}$$

The special $SL(2,\mathbb{R})$ transformation (21) will play an important role in the next section. Notice that the above solution can also be obtained by performing a special dimensional reduction of the 12-dimensional wave, which is different from the one discussed in the introduction [24].

Given the expression (17) for the position r_{min} of the wormhole we see that by changing the value of h we effectively move the position r_{min}. In other words, due to the $SL(2,\mathbb{R})$ symmetry, the position left or right from the wormhole is not determined. Any point positioned at r_{fix} with $r_{fix} > r_{min}$ is $SL(2,\mathbb{R})$ equivalent to a point positioned at r_{fix} with $r_{fix} < r_{min}$ and especially by transforming h away to $h=0$ one effectively moves towards the vacuum at $r=0$ where one reaches the flat spacetime, see equation (18).

† A similar transformation exists for the negative sign in (11).

3. *T*-duality

In the previous section we have shown that the constant part h of the harmonic function H can be removed by a special $SL(2,\mathbb{R})$ transformation. This has important consequences for the geometry described by the solution. For $h \neq 0$, the solution represents a wormhole geometry, see figure 1, whereas for $h = 0$ we obtain a flat spacetime. This flat spacetime geometry can be parametrized by different coordinate systems (r versus ρ), see equation (18). We therefore have three (locally) equivalent representations of the D-instanton:

(i) the original D-instanton solution (14);
(ii) the flat spacetime geometry (18) in the r coordinate system;
(iii) the flat spacetime geometry (18) in the ρ coordinate system.

In this section we will consider the T-dual versions of these three different representations.

Representation (i). We start with the 'standard' representation given in (14). Since there is no worldvolume direction we can only apply a T-duality in the transverse space†. It is well known that, after applying T-duality in the different transverse directions, one obtains the D-p-branes ($0 \leqslant p \leqslant 6$) with string-frame metric given by‡

$$\mathrm{d}s^2 = \frac{1}{\sqrt{H}}\,\mathrm{d}x_{p+1}^2 - \sqrt{H}\,\mathrm{d}x_{9-p}^2, \qquad \mathrm{e}^{-2\phi} = H^{(p-3)/2}, \qquad F_{0\ldots pI} = \partial_I H^{-1}, \tag{23}$$

where $I = p+1, \ldots, 9$ represents the $9-p$ transverse directions. The function H is harmonic only with respect to the transverse directions, i.e.

$$H = h + \frac{Q}{r^{7-p}}, \qquad r^2 = x_{p+1}^2 + \cdots + x_9^2. \tag{24}$$

Representation (ii). Next, we consider the T-dual version of the second representation of the D-instanton, i.e. the flat spacetime geometry in the r coordinate system, see (18). This obviously leads to the same D-brane configuration given above, but now with the constant part h in the harmonic function H being removed. On the other hand, approaching the horizon of a D-p-brane solution at $r = 0$ one can effectively ignore the constant part in the harmonic function. Therefore T-dualizing (18) (in the r-basis) yields the near-horizon geometry of the D-p-brane and therefore by the $SL(2,\mathbb{R})$ transformation (21) we effectively have approached the horizon. The solution in the string-frame is given by

$$\begin{aligned} \mathrm{d}s^2 &= \sqrt{\frac{r^{7-p}}{Q}}\mathrm{d}x_{p+1}^2 - \sqrt{\frac{Q}{r^{3-p}}}\left[\left(\frac{\mathrm{d}r}{r}\right)^2 + \mathrm{d}\Omega_{8-p}\right], \\ \mathrm{e}^{(-4/(p-3))\phi} &= \frac{Q}{r^{7-p}}, \qquad F_{0\ldots pI} = \partial_I\left(\frac{r^{7-p}}{Q}\right). \end{aligned} \tag{25}$$

Note, that for all p the spherical part of the above solutions is singular, except for $p = 3$, where one obtains the $AdS_5 \times S_5$ geometry. Thus, the D-3-brane interpolates between the Minkowskian vacuum ($r = \infty$) and the $AdS_5 \times S_5$ vacuum ($r = 0$) [15] in the same way as the D-instanton interpolates between the two Minkowskian vacua at $r = 0$ and $r = \infty$ and both regions are interchanged by the mirror transformation (15).

† Actually, following [14] one can also perform a so-called 'Hopf T-duality'. In order to perform this kind of duality, one first parametrizes the 10-dimensional transverse space of the D-instanton in polar coordinates, see (10). One next realizes the 9-sphere S^9 as a $U(1)$ bundle over CP^4 and performs a T-duality in the $U(1)$ isometry direction. One thus obtains a D-0-brane with a non-flat transverse space involving the CP^4 manifold.

‡ Note that after performing the T-duality we rotate the Euclidean space back to a Minkowskian spacetime.

Representation (iii). Finally, we consider the T-dual version of the third representation of the D-instanton, i.e. the same flat spacetime geometry as above but now in the ρ basis, see (18). More precisely, starting from the ρ basis, we first introduce Cartesian coordinates and next apply T-duality. Since we are dealing with a flat metric, the only changes are in the gauge fields. We thus arrive at the following string-frame configuration ($0 \leqslant p \leqslant 6$):

$$\mathrm{d}s^2 = \mathrm{d}x^2_{p+1} - \mathrm{d}x^2_{9-p}, \qquad \mathrm{e}^{(-4/(p-3))\phi} = Q^{(p-3)/4}\rho^{7-p}, \qquad F_{0\ldots pI} = Q^{(p-3)/4}\partial_I\rho^{7-p}, \tag{26}$$

where ρ is the radial coordinate of the transverse part. These configurations differ from the one given in (25). This was to be expected since T-duality and coordinate transformations do not commute. It is straightforward to verify that the above configuration does indeed solve the equations of motion: the $(p+1)$-form gauge fields solve the free Maxwell-like equations of motion and the expression for the dilaton is just the solution of a Laplacian equation. Notice that the relative factors between the scalar and gauge field part of the solution ensure that the energy–momentum tensor vanishes as it has to for a flat metric.

4. Supersymmetry

In this section we consider the supersymmetry of the D-instanton solution and its T-dual versions. Ignoring the gauge fields, the IIB supersymmetry rules of the gravitino and dilatino (using the Einstein metric) in Minkowskian spacetime are given by†

$$\begin{aligned} \delta\psi_\mu &= \left(\partial_\mu - \tfrac{1}{4}\omega_\mu^{ab}\Gamma_{ab} - \tfrac{1}{4}\mathrm{i}\mathrm{e}^{\phi}\partial_\mu\ell\right)\epsilon, \\ \delta\lambda &= \tfrac{1}{4}\Gamma^\mu\epsilon^\star\left(\partial_\mu\phi + \mathrm{i}\mathrm{e}^{\phi}\partial_\mu\ell\right). \end{aligned} \tag{27}$$

The supersymmetry of the D-instanton solution (11) has already been considered in [17]. After a Wick rotation to a 10-dimensional Euclidean space the supersymmetry transformations become (in Einstein frame) [17]

$$\begin{aligned} \delta\psi_\mu^{(\pm)} &= \left(\partial_\mu - \tfrac{1}{4}\omega_\mu^{ab}\Gamma_{ab} \mp \tfrac{1}{4}\mathrm{e}^{\phi}\partial_\mu\ell\right)\epsilon^{(\pm)}, \\ \delta\lambda^{(\pm)} &= \tfrac{1}{4}\Gamma^\mu\epsilon^{(\mp)}\left(\partial_\mu\phi \pm \mathrm{e}^{\phi}\partial_\mu\ell\right). \end{aligned} \tag{28}$$

Inserting the solution (11) with the '+' sign and taking into account that the Einstein metric is flat, one obtains as a solution

$$\epsilon^{(+)} = 0, \qquad \epsilon^{(-)} = \mathrm{e}^{\phi/4}\epsilon_0^{(-)} = H^{1/4}\epsilon_0^{(-)} \tag{29}$$

for a constant spinor $\epsilon_0^{(-)}$. For the negative sign in (11) one obtains a similar solution, where $\epsilon^{(+)}$ and $\epsilon^{(-)}$ are interchanged. Therefore, the D-instanton solution generically breaks $\frac{1}{2}$ of the supersymmetry. The same is true for the $SL(2,\mathbb{R})$ transformed solution (22) since the supersymmetry rules (28) are $SL(2,\mathbb{R})$-covariant.

Next, we discuss the supersymmetry in the two asymptotic flat regions. First, in the $r = \infty$ vacuum obviously all supersymmetry is restored ($\phi = \ell =$ constant). Requiring that the gravitino variation vanishes we find that the spinors in the limit $r \to 0$ behave like $\epsilon^{(\pm)} \sim r^2\epsilon_0^{(\pm)}$. Thus, both spinors $\epsilon^{(\pm)}$ vanish like r^2 and as a consequence both dilatino variations $\delta\lambda^{(\pm)}$ vanish identically. We conclude that, *in both asymptotic regions of the D-instanton solution ($r = \infty$ and $r = 0$) we have a restoration of unbroken supersymmetry*.

† These supersymmetry rules, using an $SU(1,1)$-basis, have been given in [21]. Here we use the $SL(2,\mathbb{R})$-covariant form of these rules, as given in [25].

Next, we consider the supersymmetry of the T-dual versions of the D-instanton. After T-duality and rotating back to Minkowskian signature the relevant part of the supersymmetry variations in the string frame becomes ($0 \leqslant p \leqslant 6$) [26]†

$$\begin{aligned} \delta\psi_\mu &= \partial_\mu\epsilon - \frac{1}{4}\omega_\mu^{ab}\Gamma_{ab}\epsilon + \frac{(-)^p}{8(p+2)!}\mathrm{e}^\phi(F\cdot\Gamma)\Gamma_\mu\epsilon'_{(p)}, \\ \delta\lambda &= \frac{1}{4}\Gamma^\mu(\partial_\mu\phi)\epsilon^\star + \frac{3-p}{16(p+2)!}\mathrm{e}^\phi(F\cdot\Gamma)\epsilon'^\star_{(p)}, \end{aligned} \tag{30}$$

where $(F\cdot\Gamma) = F_{\mu_1\cdots\mu_{p+2}}\Gamma^{\mu_1\cdots\mu_{p+2}}$ and the parameters $\epsilon'_{(p)}$ are defined in table 1.

Table 1. Definition of spinors $\epsilon'_{(p)}$. The table gives the definition of the spinor parameters $\epsilon'_{(p)}$ occurring in the supersymmetry transformations (30) in terms of the supersymmetry parameter ϵ.

p	$\epsilon'_{(p)}$ (IIA)	p	$\epsilon'_{(p)}$ (IIB)
0	ϵ	1	$\mathrm{i}\epsilon^\star$
2	$\gamma_{11}\epsilon$	3	$\mathrm{i}\epsilon$
4	ϵ	5	$\mathrm{i}\epsilon^\star$
6	$\gamma_{11}\epsilon$	—	—

Substituting the D-p-brane solutions (23) or the solutions (25) into the supersymmetry rules (30) we find as a solution for the vanishing of these supersymmetry variations

$$\epsilon + \Gamma_{0\cdots p}\epsilon'_{(p)} = 0, \qquad \epsilon = H^{-1/8}\epsilon_0. \tag{31}$$

This shows that the solutions (23) and (25) for all p have half of unbroken supersymmetry. In addition, for special cases we have a restoration of unbroken supersymmetry [27]. For the D-p-brane solutions (23) this is the case in the asymptotic vacuum ($r = \infty$) and for the 3-brane case this also happens near the horizon ($r = 0$). For the solutions (25) we only have unbroken supersymmetry for $p = 3$ in which case the solution has the $AdS_5 \times S_5$ geometry.

Finally, we consider the supersymmetry of the solutions (26). Although it seems to be natural to take this coordinate system, it has important consequences for the supersymmetry. Considering the 'worldvolume' components (= isometry directions) of the gravitino variation (30), we find that for $0 \leqslant p \leqslant 6$ all supersymmetries are broken. The technical reason for this is that in the worldvolume components of the gravitino variation the $\partial\epsilon$ vanishes since the Killing spinor does not depend on the worldvolume directions and furthermore without gravity we have that ω_μ^{ab} vanishes as well. However, the field strength F is non-trivial and this leads to a complete breaking of the supersymmetry. This result might be surprising since the original ($p = -1$) solution allowed $\frac{1}{2}$ of the unbroken supersymmetry. However, this case is special since, although there is no gravity, neither are there any 'worldvolume' components of the gravitino variation to consider.

5. Intersections

Turning on and off the constant parts in the harmonic functions also has important consequences for intersections. Let us start by presenting a systematic method of constructing intersections starting from the D-instanton. The idea is to solve the scalar

† Actually, for $p = 3$, due to the self-duality condition of the 5-form field strength, one should include an extra factor of $\frac{1}{2}$ in front of the $\Gamma \cdot F$ term in the gravitino rule. We thank Kostas Sfetsos for pointing this out to us.

field equations for ϕ and ℓ, that determine the instanton, in the background of a further brane. We are interested in threshold bound states (intersections) and therefore the gauge fields are given by independent harmonic functions (or charges). For the instanton this means that ℓ has to be independent of the background brane and given the equation of motion for ℓ one realizes that only the harmonic function of a 3-brane drops out†.

In the Einstein equations the instanton does not contribute to the energy–momentum tensor (due to the ansatz (11)) and thus, they are solved by the D-3-brane metric, see equation (23). Next, using the instanton ansatz (with harmonic function H_1) the two scalar field equations in (11) become

$$0 = \partial_\mu\big(\sqrt{|g|}g^{\mu\nu}\partial_\nu \mathrm{e}^{\phi}\big), \tag{32}$$

where we have to insert the D-3-brane metric (with harmonic function H_2) in the Einstein frame. For this brane, however, the Einstein and string metric are the same and we find $\sqrt{|g|}g^{\mu\nu} = \delta^{\mu\nu}$. Hence we obtain a harmonic equation

$$\mathrm{e}^{-\phi} = \pm\ell + \alpha_1 = \frac{1}{H_1}. \tag{33}$$

Notice that *only* for the D-3-brane (and D-7-brane) background do we get a flat Laplacian, for all other branes the scalar fields are more complicated, indicating a violation of the harmonic superposition rule [28]. By this procedure, one naturally obtains the two known families of intersections of two branes, which have four (or eight) relative-transverse coordinates. Only in the first class (corresponding to the D-3-brane case) do both harmonic functions depend on the overall transverse coordinates.

We thus obtain the intersection‡ $(-1)\times 3$, from which we can obtain all other intersecting brane configurations by U-duality. Approaching the horizon the 4D relative transverse space factorizes, i.e. $\mathbb{M}_{10} \to \mathbb{M}_6 \times \mathbb{E}_4$. We therefore end up effectively with a 6D theory, with branes coming from 10D intersections. Like in 10 dimensions also in six dimensions we have just one non-singular object, the self-dual string. Only for this object does the spacetime factorize further into $AdS_3\times S_3\times\mathbb{E}_4$. The 10D configurations giving this geometry are 1×5, 2×4 and 3×3.

Like for the single branes one can also reach the near-horizon geometry by an $SL(2,\mathbb{R})$ transformation. Applying the special $SL(2,\mathbb{R})$ transformation (21) to the fields given in (33) one turns off the constant part of H_1. This $SL(2,\mathbb{R})$ transformation does not effect the other fields corresponding to the D-3-brane since the D-3-brane is $SL(2,\mathbb{R})$ invariant. In a second step, using U-duality, we turn off the constant part of the second harmonic function H_2. By this procedure we obtain a 10D spacetime factorizing into $\mathbb{M}_{10} = \mathbb{M}_6\times\mathbb{E}_4$, where $\mathbb{E}_4$ denotes the relative transverse part. If the branes intersect over a 0- or 2-brane the $\mathbb{M}_6$ part does not factorize further, but if it intersects over a string the spacetime factorizes further into $\mathbb{M}_{10} = AdS_3\times S_3\times\mathbb{E}_4$.

As an example, consider the case 3×3 which is given by

$$\mathrm{d}s^2_{3\times 3} = \frac{1}{\sqrt{H_1H_2}}\big(\mathrm{d}t^2 - \mathrm{d}z^2\big) - \sqrt{H_1H_2}\,(\mathrm{d}x_m)^2 - \sqrt{\frac{H_1}{H_2}}\,\big(\mathrm{d}x_6^2 + \mathrm{d}x_7^2\big) - \sqrt{\frac{H_2}{H_1}}\,\big(\mathrm{d}x_8^2 + \mathrm{d}x_9^2\big), \tag{34}$$

where z denotes the direction of the common string, $x_6, \ldots, x_9$ are the relative transverse coordinates and the harmonic functions are given by $H_i = 1 + q_i/r^2$ $(i = 1, 2)$. Using

† A D-7-brane background is also possible, but in that case the harmonic functions of the two intersecting branes depend on different (relative transverse) coordinates. We will not consider this possibility further here.

‡ After the submission of this work we learned that a configuration describing a localized D-instanton solution within a D-3-brane system has been obtained in [29].

the U-duality chain described in section 3, locally for any radius r this configuration is equivalent to

$$\mathrm{d}s^2_{3\times 3} = \frac{r^2}{\sqrt{q_1 q_2}}\left(\mathrm{d}t^2 - \mathrm{d}z^2\right) - \sqrt{q_1 q_2}\left(\frac{\mathrm{d}r}{r}\right)^2 - \sqrt{q_1 q_2}\,\mathrm{d}\Omega_3 - \sqrt{\frac{q_1}{q_2}}(\mathbb{E}_2) - \sqrt{\frac{q_2}{q_1}}(\mathbb{E}_2) \qquad (35)$$

which is $AdS_3 \times S_3 \times \mathbb{E}_4$.

It is not difficult to add further branes to the intersecting configuration. We will start with four intersecting branes and afterwards, by truncation, we will consider the triple intersections as a special subclass. The simplest way is to start with the 3×3 configuration (34), is to add a wave W along the common string direction z and to insert a KK-monopole KK with its Taub-NUT part in the four-dimensional overall transverse space. This intersection is U-dual to $(-1) \times 3 \times W \times KK$ and because the 3-brane as well as the wave and KK monopole are $SL(2,\mathbb{R})$ invariant, we can again use the $SL(2,\mathbb{R})$ rotation (21) to turn off the constant part of the harmonic function describing the (-1)-brane and subsequently we can do the same for all other branes as well. For the triple intersections we have to turn off either the wave, obtaining the 5D string, or the KK monopole yielding the 5D black hole.

In complete analogy to the double intersections discussed before, the geometry factorizes. Keeping at least three overall transverse dimensions the triple intersections factorize into $\mathbb{M}_{10} = AdS_3 \times S_2 \times \mathbb{E}_5$ for the 5D string, $\mathbb{M}_{10} = AdS_2 \times S_3 \times \mathbb{E}_5$ for the 5D black hole and finally the quadruple intersection corresponding to the 4D black hole factorizes into $\mathbb{M}_{10} = AdS_2 \times S_2 \times \mathbb{E}_6$. This leads exactly to the non-singular near-horizon geometries considered in [10, 27].

6. Conclusions

In this paper we have employed, via the D-instanton, the $SL(2,\mathbb{R})$ duality symmetry of the type IIB superstring theory to turn off the constant part in the harmonic function describing brane solutions. This is in analogy to the large-N limit of the D-3-brane [7], in both cases the constant part is effectively neglected. This process has important consequences for the geometry of the solution. The D-instanton geometry is a wormhole that interpolates between two asymptotic flat space vacua and there is a mirror map (15) that transforms both vacua into each other. As one can see in figure 1 if the constant part h of the harmonic function vanishes, the throat shrinks and the minimum moves towards infinity, i.e. the vacua at infinity disappear and the throat closes. One ends up with a flat spacetime, both in Einstein as well as in the string frame metric. Of course, the same also happens in the mirror base ρ defined below (18). Notice, this situation (i.e. vanishing h) is reached simply by employing a symmetry of the theory, namely the type IIB $SL(2,\mathbb{R})$ duality transformation (21). We discussed three topologically different representations of the D-instanton (one wormhole and two flat-space descriptions), which are $SL(2,\mathbb{R})$-dual to each other.

By applying a standard T-duality one can convert the D-instanton into all other D-branes. Starting with the case of non-vanishing h one obtains the standard branes, but if $h = 0$ we had to distinguish between the two mirror bases. In one case one gets the near-horizon geometry of the D-branes, but in the other case one obtains flat spacetime with non-trivial gauge fields and dilaton. Investigating the supersymmetry we found that in the first case $\frac{1}{2}$ of supersymmetries are broken and for the 3-brane all supersymmetries are restored, but for the latter case (flat spacetime) all supersymmetries are broken. The supersymmetry breaking in the latter case can be understood due to the coordinate transformation (mirror map).

It is straightforward to apply the same procedure discussed above to intersecting branes and we discussed a procedure to construct intersecting branes 'out of the D-instanton'. Thus, we could turn off all constant parts by S-dualizing the D-instanton part. For all brane configurations with a non-singular horizon this has the consequence that the spacetime factorizes into the structure $AdS_p \times S_q \times \mathbb{E}_r$. At this point we should stress, that in order to keep a well defined low-energy limit we should keep all charges sufficiently large and a certain hierarchy to keep all higher derivative corrections under control.

Summarizing, we are facing an intriguing situation. In this work we have shown that by applying T- and S-duality one can move effectively towards the horizon, where for the non-singular cases the supersymmetry is enhanced. In addition, in the case of the D-instanton we changed the topology, simply by duality. But, as stressed at many places, all discussions are valid only locally. We do not yet have a complete picture about what happens globally, but there are some interesting points to mention. A first hint comes from the action. The ℓ field is the gauge field for the D-instanton and we see, that if one turns off the constant part of the harmonic function the gauge coupling (which is the $\mathrm{e}^{2\phi}$ factor in front of the ℓ kinetic term, see (7)) vanishes at infinity, indicating a phase transition. Geometrically, in this phase transition one asymptotic flat region of the D-instanton disappears or for the non-singular D-brane configurations the spacetime factorizes. This change on the boundary is also visible in the D-instanton action, which appears by integrating out the boundary terms in (7) (see [17]), and is given by

$$S_{\partial M} = Q\big(\ell \pm \mathrm{e}^{-\phi}\big)_{\infty}\Omega_9. \qquad (36)$$

Because $\big(\mathrm{e}^{-\phi}\big)_{\infty} = 1/h$ and if $h \to 0$, this term becomes infinite. This was expected, because instantons describe tunnelling processes between the two asymptotic vacua and they are exponentially suppressed in the weak coupling limit, which is given by $h = \big(\mathrm{e}^{\phi}\big)_{\infty} = g \to 0$.

The procedure discussed in this paper supports the idea of the holographic principle [30]. Taking any non-singular brane configuration, we were arguing that any (finite) point in spacetime is U-dual to the geometry $AdS_q \times S_p \times \mathbb{E}_r$. This space can be reduced to the anti-de Sitter space, which is fixed by a (conformal) field theory living on the boundary [7, 11]. This means that the physics at any (finite) point is determined by the boundary and not by local degrees of freedom.

Finally, in this work we used the 'standard' D-instanton solution which has a flat transverse space. As suggested in [13, 23] there exists a more general class of brane solutions, where the spherical part (S_9 in our case) is replaced by a general Einstein space, see the footnote below (10). By doing this, in general, one breaks more supersymmetry and the spherical parametrization we used in this work is maximal supersymmetric. The orbifold construction discussed in [12] is one example of this construction. It would be of interest to extend the results of this work to these more general D-instanton solutions.

Acknowledgments

EB thanks S-J Rey for stimulating discussions and the physics department of Seoul National University for its hospitality. KB thanks the physics department of Groningen university for its hospitality. This work is supported by the European Commission TMR programme ERBFMRX-CT96-0045, in which EB is associated to the university of Utrecht. The work of KB is supported by the DFG.

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